

# Luttinger Liquid at the Edge of a Graphene Vacuum

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We demonstrate that an undoped two-dimensional carbon plane (graphene) whose bulk is in the integer quantum Hall regime supports a non-chiral Luttinger liquid at an armchair edge. This behavior arises due to the unusual dispersion of the non-interacting edges states, causing a crossing of bands with different valley and spin indices at the edge. We demonstrate that this stabilizes a domain wall structure with a spontaneously ordered phase degree of freedom. This coherent domain wall supports gapless charged excitations, and has a power law tunneling  $I - V$  with a non-integral exponent. In proximity to a bulk lead, the edge may undergo a quantum phase transition between the Luttinger liquid phase and a metallic state when the edge confinement is sufficiently strong relative to the interaction energy scale.

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*Introduction* – Two-dimensional carbon sheets [1], known as graphene, are emerging as one of the most exciting new systems supporting the quantum Hall effect. This material is different than more standard two-dimensional electron gases (2DEGs) because in the absence of a magnetic field, the single particle spectrum is linear in the vicinity of two inequivalent points in the Brillouin zone. The low-energy states near these points are described by the Dirac equation [2], and in a strong magnetic field, the quantum Hall steps that emerge are shifted relative to standard 2DEGs [3, 4]. This effect arises because the spectrum of the Dirac equation with a magnetic field has doubly degenerate Landau levels (LL) for each spin, with one pair at zero energy, half of which are filled in the nominally undoped situation. Thus filling the Landau states to reach the first quantum Hall plateau requires only half as many electrons as needed to reach the subsequent plateaus, shifting the step pattern.

In an undoped standard 2DEG system, there is little interesting electron physics because the filled valence states are far below the chemical potential in practical situations. By contrast, the partially filled LL at zero energy in graphene allow for interesting low-temperature physics even in this nominal “vacuum”. When interactions are included, the half-filled zero energy states represent a multicomponent system, which in the absence of spin or valley splitting potentials spontaneously polarizes due to exchange [5]. In practice, it is spin-polarized in the bulk due to the Zeeman splitting. As we show below, the vacuum is thus a quantum Hall ferromagnet, with an associated low-energy spin-wave.

In addition to these surprising bulk properties, graphene also has an unusual edge structure even in a non-interacting picture: the lowest LL has half as many edge states as the higher LLs [6]. In this work, we demonstrate a remarkable effect when the edge structure and the quantum Hall ferromagnetism are both taken

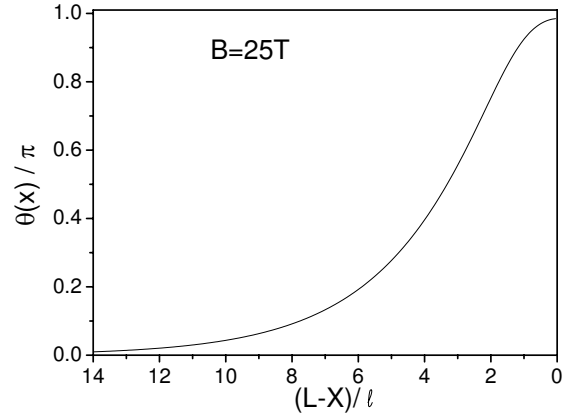


FIG. 1: Example of a domain wall configuration. See text.

into account. Under appropriate circumstances undoped graphene forms a coherent domain wall (DW) between the spin polarized state in the bulk and an unpolarized region at the edge. The low-energy theory of this DW has a U(1) symmetry with a Luttinger liquid Hamiltonian [7]. More specifically, the DW may be described by a variational wavefunction of the form

$$|\Psi\rangle = \prod_{X < L} \left( \cos \frac{\theta(X)}{2} + \sin \frac{\theta(X)}{2} e^{i\phi} C_{-\downarrow X}^\dagger C_{+\uparrow X} \right) |\text{Vac}\rangle, \quad (1)$$

where  $C_{+\uparrow X}^\dagger$  and  $C_{-\downarrow X}^\dagger$  create electrons in the two relevant levels,  $X$  is the guiding center quantum number with allowed values up to the edge at  $L$ ,  $|\text{Vac}\rangle$  denotes the bulk undoped graphene state (i.e., vacuum) which is partially polarized since the two spin up lowest LLs are fully occupied, and  $\theta(X)$  and  $\phi$  are variational parameters. An example of  $\theta(X)$  found by minimizing the effective energy functional is illustrated in Fig. 1. The energy of

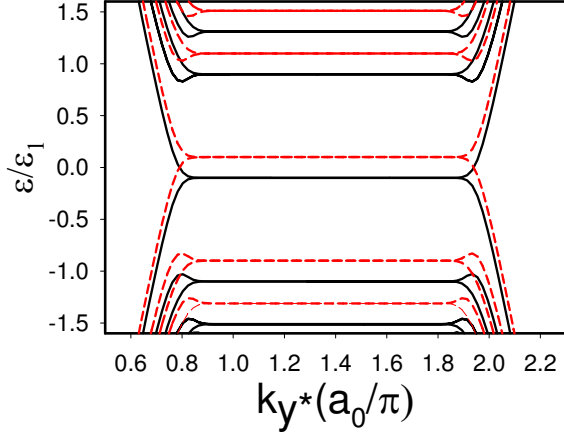


FIG. 2: (Color online) Energy bands for electrons in a graphene ribbon with armchair edges in a magnetic field from tight binding calculations.  $a_0$  = lattice constant, unit of energy  $\varepsilon_1 = \sqrt{2}\gamma a_0/\ell$ .  $k_y = X/\ell^2$  with  $X$  the guiding center coordinate.  $B=100\text{T}$ , ribbon width is  $460\text{\AA}$ .

the state is independent of  $\phi$ , indicating a spontaneously broken symmetry in the DW groundstate, with an associated gapless collective mode which may be understood as states in which  $\phi$  has a spatial gradient [8, 9, 10]. Gradients in  $\phi$  carry a charge density, and a full  $2\pi$  rotation contains a single electron above the vacuum [11, 12]. Thus this system supports *gapless* charged excitations.

This coherent DW may be probed by tunneling into it from a bulk metallic lead [13]. For a standard 2DEG, in the undoped case the system trivially behaves as an insulator, and for integer quantum Hall states one finds a metallic response [13]. For the coherent DW, we expect a power law tunneling  $I - V$ , with exponent determined by the pseudospin stiffness and the strength of the confinement potential. This is quite different than standard quantum Hall edges, where the tunneling exponent is thought to be set by the bulk filling factor [14]. By varying the strength of the electron-electron interaction (for example, by a screening gate) or the edge confinement potential, one can vary the tunneling exponent, and in principle may drive the system through a quantum phase transition in which the tunneling perturbation becomes relevant in the renormalization group sense. This presumably drives the system into a metallic state with a linear  $I - V$ .

We now provide details of these results.

*Pseudospin Ferromagnetism in Graphene* – The non-interacting spectrum of a graphene ribbon in a magnetic field with armchair edges is illustrated in Fig. 2 [6]. The eigenstates are characterized by a quantum number  $X$

which specifies the center of the wavefunction along the direction perpendicular to the edge, and spin ( $\sigma = \pm\frac{1}{2}$ ) and valley indices ( $\tau = \pm 1$ ) for a total of four degenerate Landau bands inside the bulk of the system. In the undoped system, all the negative energy states and two of the zero energy states [15] are filled at zero temperature. In what follows, we will ignore the LLs well away from zero energy, since these are either completely filled or empty and will not affect the physics described here.

Retaining just the four lowest Landau levels (LLLs) near zero energy, apart from constant terms the Hamiltonian may be written as

$$H = \sum_{\tau, \sigma, X} \left[ -E_z \sigma C_{\tau \sigma X}^\dagger C_{\tau \sigma X} + \Delta(X) \tau C_{\tau \sigma X}^\dagger C_{\tau \sigma X} \right] + \frac{N_\phi^2}{2S} \sum_{\sigma, \sigma', \tau, \tau', \mathbf{q}} e^{-\frac{q^2 \ell^2}{2}} V_{\mathbf{q}} \rho_{\tau, \sigma}(-\mathbf{q}) \rho_{\tau', \sigma'}(\mathbf{q}). \quad (2)$$

In this expression,  $E_z = g\mu_B B$  is the Zeeman coupling,  $\Delta(X)$  is the energy splitting produced by the edge [6],  $N_\phi/S$  is the number of flux quanta per unit area passing through the system, and  $V_{\mathbf{q}} = \frac{2\pi e^2}{\epsilon_0 q}$  is the Coulomb interaction. Note that we have assumed an SU(4) symmetric form for the interaction. Although not exact this should be an excellent approximation for LLL states provided the magnetic length is much larger than the lattice constant [2]. Finally, the LLL density operators have the form  $\rho_{\tau, \sigma}(\mathbf{q}) = \frac{1}{N_\phi} \sum_X e^{-\frac{i}{2} q_x (2X + q_y)} C_{\tau \sigma X}^\dagger C_{\tau \sigma X + q_y}$ .

The vacuum state (i.e., undoped groundstate) involves filling two of the four LLL. For  $\Delta = E_z = 0$ , any two orthogonal states will yield the same energy. In the bulk (where  $\Delta = 0$ ) the Zeeman coupling breaks this symmetry, and in the Hartree-Fock approximation the vacuum state may be written in the form

$$|\text{Vac}\rangle = \prod_X C_{\uparrow X}^\dagger C_{\downarrow X}^\dagger |n < 0\rangle, \quad (3)$$

where  $|n < 0\rangle$  denotes the state with all the negative energy levels filled. Our vacuum is thus ferromagnetic. The energy per particle of the ground state is  $-E_z + \frac{1}{2}\Sigma$  with  $\Sigma = -\sqrt{\frac{\pi}{2}} \frac{e^2}{\epsilon_0 \ell}$  the electron self-energy. Note that this energy scale is much larger than  $E_z$ , and that the primary role of  $E_z$  is to pick out which two states are actually occupied.

The ferromagnetic vacuum supports four collective neutral excitations, which may be approximately generated by applying the operators

$$\rho_{\tau, \tau', \uparrow}(\mathbf{q}) = \frac{1}{N_\phi} \sum_X e^{-\frac{i}{2} q_x (2X + q_y)} C_{\tau \downarrow X}^\dagger C_{\tau' \uparrow X + q_y} \quad (4)$$

to  $|\text{Vac}\rangle$ . Each of these involves a spin flip, and two of them also include a valley density wave. Because of the SU(4) symmetry of the interactions, all four excitations are degenerate with energy at long wavelengths  $\omega_0(q) \approx$

$2E_z + 4\pi\rho_s q^2 \ell^2$ , with  $\rho_s = 1/16\sqrt{2\pi}$  in units of  $e^2/\epsilon_0\ell$ , and  $\ell = \sqrt{c/eB}$  is the magnetic length. This is precisely the form one expects for a ferromagnet in the presence of a Zeeman coupling.

It is interesting to notice that due to the spin-charge coupling inherent in a LL, doping the system could degrade the spin polarization [17]. Previous estimates [18] suggest that  $E_z$  is too large in the bulk for this to occur. However, near the edge where the non-interacting levels cross, we now demonstrate that such effects are crucial for obtaining a correct description of the lowest energy states of the system.

*Coherent Domain Wall at the Edge* – Fig. 2 illustrates the behavior of the non-interacting levels with armchair edges. Because of the Zeeman splitting, a hole-like spin-up state necessarily crosses an electron-like spin-down state [6, 19]. We note that the  $\tau = +, -$  states near the edge do not represent states purely in either valley, as they are admixed by the boundary condition for an armchair edge [6]. Because of its effective spin stiffness, the groundstate does not adhere to the lowest energy single particle state, but rather generates a DW to make the spatial transition. Eq. 1 describes such a DW when  $\theta(X)$  passes from 0 to  $\pi$ . The energy of this state can be shown to have the form [20, 21]

$$E \simeq \pi\ell^2\rho_s \sum_{X < L} \left( \frac{d\theta}{dx} \right)^2 + \sum_{X < L} [\Delta(X) - E_z] \cos\theta(X),$$

for an edge at  $x = L$ , provided  $\theta(X)$  does not vary rapidly on the scale of  $\ell$ . This sine-Gordon-like energy functional may be straightforwardly minimized numerically [20]. A typical example of a DW solution is illustrated in Fig. 1.

Because the energy of  $|\Psi\rangle$  is independent of  $\phi$ , it is immediately clear that this represents a broken symmetry state which must support a gapless mode. The DW is effectively a one-dimensional easy-plane ferromagnet, which may be described by an effective action

$$S_0 = \int d\tau dy \left[ \frac{\Gamma}{2} m(y, \tau)^2 + \frac{\tilde{\rho}}{2} \left( \frac{\partial\phi}{\partial y} \right)^2 + i m(y, \tau) \left( \frac{\partial\phi}{\partial\tau} \right) \right] \quad (5)$$

where  $\tau$  is imaginary time. The field  $m$  has its origin in the variable  $\theta$ , and may be understood as the position of the center of the DW. The constants  $\Gamma$  and  $\tilde{\rho}$  may be estimated from  $\theta(X)$  and  $E_z - \Delta(X)$  using spin-wave theory [10, 20]. Details of this will be published elsewhere.

Because  $\theta(X)$  passes from 0 to  $\pi$  over the width of the domain wall, any configuration of  $\phi(y)$  that passes from 0 to  $2\pi$  represents a state in which the sphere of allowed directions of an effective spinor constructed from the two states  $(+, \uparrow)$  and  $(-, \downarrow)$  is covered precisely once. In this situation the state carries an excess or deficit of one electron relative to the groundstate [11, 12]. Moreover, the twist in  $\phi$  may be spread out over the entire length of the domain wall, so that charge may be introduced or removed with arbitrarily low energy, in contrast

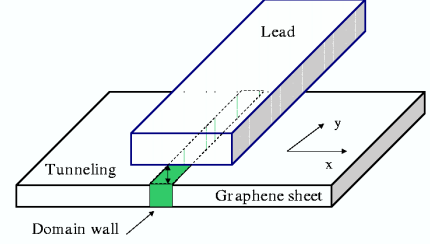


FIG. 3: (Color online) Geometry for tunneling into the coherent domain wall.

to the gap present for charged excitations in the bulk. As we now demonstrate, this has important implications the tunneling into the edge of graphene vacuum.

*Tunneling from a Metallic Electrode* – The geometry for tunneling into the DW is illustrated in Fig. 3. We model the lead as a simple Fermi liquid, with action

$$S_{Lead} = -\frac{1}{\beta} \sum_{\omega_n, n, k_z, k_x} [E_n(k_z) - \mu + i\omega_n] a_{nk_y k_z}^*(\omega_n) a_{nk_y k_z}(\omega_n),$$

where  $\beta$  is the inverse temperature and  $\mu$  the chemical potential. The Grassman variable  $a_{nk_x k_z}$  corresponds to the destruction operator for a lead electron in the  $n$ th Landau level state  $\Phi_{nk_y k_z} = \frac{1}{\sqrt{L_y L_z}} e^{ik_y y + ik_z z} \varphi_n(x - k_y \ell^2)$  with  $\varphi_n$  a harmonic oscillator state, and  $E_n(k_z) = (n + 1/2)\omega_c + k_z^2/2m$ ,  $\omega_c = eB/mc$  being the cyclotron frequency. The tunnel coupling between the lead and reservoir takes the form

$$S_{tun} = \frac{1}{\beta} \sum_{\omega_n} \sum_{n k_y} \int dy \chi_{nk_y}(y) a_{nk_y k_z}^*(\omega_n) \psi(y, \omega_n) + h.c.$$

where  $\psi$  is the annihilation operator for an electron in the domain wall, which we have taken to be located at  $x = z = 0$ , and  $\chi_{nk_y}(y) = \frac{t}{\sqrt{L_y}} e^{-ik_y y} \varphi_n(-k_y \ell^2)$ . Using standard bosonization [7], one may write the fermion operator as  $\psi(y, \tau) \sim e^{-i\phi(y, \tau)} e^{i2\pi \int_{-\infty}^x dy' m(y', \tau)}$ . After tracing out the lead degrees of freedom, one finds the partition function may be written in the form  $Z = \int \mathcal{D}^2 a \mathcal{D}\phi \mathcal{D}m e^{-S_0 - S_{Lead} - S_{tun}} \propto \int \mathcal{D}\phi \mathcal{D}m e^{-S_0 - \tilde{S}}$ , with

$$\tilde{S} = -\frac{1}{\beta} \sum_{\omega_n} \sum_{n k_y k_z} \int dy_1 dy_2 \frac{\chi_{nk_y}(y_1) \chi_{nk_y}^*(y_2) \psi^*(y_1, \omega_n) \psi(y_2, \omega_n)}{i\omega_n - \mu + E_n(k_z)}.$$

Without qualitatively changing the result, one can restrict the LL sum to just the LLL. By noting that

$\sum_{k_y} \chi \chi^*$  is strongly peaked at  $y_1 = y_2$ ,  $\tilde{S}$  may be recast in the form

$$\tilde{S} = -t^2 \int_0^\beta d\tau_1 d\tau_2 \int dy \psi^*(y, \tau_1) K(\tau_1 - \tau_2) \psi(y, \tau_2). \quad (6)$$

Taking the zero temperature limit, one may show [20]  $K \sim 1/(\tau_1 - \tau_2)$  for large  $|\tau_1 - \tau_2|$ .

Our first question is whether the presence of  $\tilde{S}$  qualitatively affects the state of the system; i.e., is it a relevant operator. A perturbative renormalization group (RG) analysis may be applied to  $\tilde{S}$  [20], leading to the result

$$\frac{dt^2}{d\ell} = -(\kappa - 2)t^2$$

with the anomalous dimension  $\kappa = (x + 1/x)/2$ , and  $x = 4\pi\sqrt{\tilde{\rho}/\Gamma}$ . Estimates of  $\tilde{\rho}$  and  $\Gamma$  using a spin-wave approach [20] yield  $\kappa \approx 6.8, 6.0$ , and  $5.3$  for  $B = 15\text{T}$ ,  $25\text{T}$ , and  $45\text{T}$ , respectively. This indicates that under usual conditions,  $\tilde{S}$  is irrelevant, and the DW remains in a Luttinger liquid phase in spite of the coupling to the lead. The physical reason for this is that the pinning energy of the domain wall is small compared to the stiffness of the phase angle, because the Zeeman energy which ultimately creates the DW is quite small compared to the electron-electron energy scale. This suggests that enhancing the relative pinning energy can drive the system into a state in which  $\tilde{S}$  is relevant, perhaps by judicious use of a gating geometry. In this situation the coupling to the metallic lead becomes important in the low-energy physics, and presumably a current injected into the domain wall will behave metallicity.

The irrelevance of  $\tilde{S}$  under ambient conditions indicates that we can treat it perturbatively. The tunneling current is then a convolution of the spectral functions for the lead and the domain wall, separated in frequency by  $eV$ , with  $V$  the potential difference between the two systems [7, 22]. The latter spectral function is the Fourier transform of the domain wall correlation function  $\langle \psi(y, \tau) \psi^\dagger(y, 0) \rangle \sim 1/\tau^\kappa$ . The power law dependence of this Green's function leads to an anomalous power law  $I - V$ , a well-known signature of Luttinger liquids. It should be emphasized that this differs considerably from edge state tunneling in standard 2DEGs, where the exponent is set by the bulk filling factor. The continuously varying exponent we find for the edge of the graphene vacuum are a consequence of the ferromagnetic nature of the undoped state, and its unusual crossing of particle- and hole-like edge states.

In summary, we have studied the armchair edge of graphene in its undoped state. We demonstrated the ferromagnetic nature of the system due to interactions, and showed how this leads to a coherent domain wall

due to a crossing of the non-interacting edge states. The domain wall can be probed by coupling it to a normal lead, and we found this supports a power-law tunneling current and the possibility of a quantum phase transition into a metallic state.

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